sults of our study also made its economic worth clear.

The Spanish Peak source is nestled in an area of superb esthetic value, and its development could easily impair this value. Fortunately, its economic worth is relatively small, and it is therefore not a candidate for development.

The Plumas County source is close to another transportation corridor and since it is only marginally important to the Quincy-Oroville corridor, the county decided to determine its economic worth to both corridors. Prior to our study, the county had no way to do this and, therefore, no way to decide this issue.

The districts are using the study as the basis for creating a minerals management policy and for negotiating with private owners to acquire desirable mineral sources. Two other district rangers asked us to apply the model to help them resolve similar problems. In their view, the results could also be used as a basis for cost-sharing agreements when one agency supplies material to another. The approach could also be applied to other materials, such as logging slash that is to be transported to central disposal sites. ■

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Literature Cited

AMERICAN ASSOCIATION OF STATE HIGHWAY OFFICIALS. 1972. Interim guide for the design of pavement structures. Washington, D.C.

KIRBY, M., and R. J. LOWE. 1975. Optimal policies for transporting rock aggregate to low volume roads. P. 296-301 *in* Proc. Workshop on Low-Volume Roads, Transportation Research Board, NAE-NRC. Boise, Idaho.

USDA FOREST SERVICE. 1973. The timber transport model, chapter III of transportation network analysis user's guide. Transportation Analysis Group, U.S. For. Serv. Berkeley, Calif.

Standardization of Growth Curves

Boris Zeide

ABSTRACT—It is shown that precisely two points are necessary and sufficient to determine any stand growth curve. Accordingly, the diversity in growth curves of forests throughout the world is reduced to a few types that accurately describe growth of any coniferous or hardwood high forest stand.

American foresters, as well as their colleagues throughout the world, usually determine the site index of a given stand by the value of height at one certain age. This concept was abstracted from extensive practice and can be referred to as the principle of one point. It implies that only one height/age pair, or one point on a site index curve, is necessary and sufficient in order to select the appropriate curve.

The Principle of One Point

During the late 1800s and early 1900s there was a tendency toward unlimited application of the principle. A Russian forester, A. V. Turin (1913, p. 96) wrote, for example: "The normal pine stands with equal heights at a certain age had the same growth in the past and will have the same growth in the future regardless of where they grow." General yield tables, prepared at that time in various countries through generalization of local tables, manifested this tendency. The most general standardization of site index curves based on the principle of one point was made in Russia. In 1911 one set of curves for all high forests, regardless of location and species, was put into practice and is still in use even though the variety of forests in Russia is comparable to that in America.

An unwarranted expansion of the principle inevitably causes significant error. In nature, the growth of different stands may often be the same at a certain age and markedly different at other ages. A number of attempts have therefore been made to determine the limits within which the principle works properly. For this reason many site index curves specified for species, area, density, and rate of early growth have been prepared.

American foresters comprehended the limitation of the one-point principle and pragmatically exercised more restraint than the Europeans. In the first American textbook on forest mensuration Graves (1906, p. 318) wrote: "It is a good rule in this country to make separate yield tables for different forest regions." Attempts to prepare yield tables for relatively wide regions, such as those made by Meyer (1937) for sitka spruce and western hemlock, have been rejected (Barnes 1962). A widespread tendency in American forestry is to recognize even slight differences in growth by preparing numerous sets of site index curves which often coincide with each other.

As far back as 1931 Bull criticized anamorphic site index curves, another manifestation of the one-point principle. Anamorphic curves never intersect each other, and therefore one point is always sufficient to select the appropriate curve. Bull (1931) proposed the use of polymorphic site index curves. This idea was further developed by Stage (1963). Many works based on the Bull-Stage idea followed.

Polymorphic curves are based on a concept other than the principle of one point, since the curves may intersect each other. The implication is that one point is not sufficient to determine the curve suitable for a given stand.

The Principle of Two Points

When we are not confident that a given site index curve fits a certain stand we must find the stand height at a number of ages or, in other words, compare a number of the curves' points with actual growth. What is the minimum number of points necessary? The polymorphic concept does not provide the answer. It merely states that one point is not enough.

The simplest assumption for this case is that two points are needed. This assumption has been mentioned in previous works (Stage 1963, Bailey and Clutter 1974). In order to test it, 299 series of height growth data were taken from 118 yield tables for coniferous and broadleaved high forest stands. These tables were from many countries, including America.

The procedure was as follows: all series were normalized in such a way that the values would be 1.00 (or 100) at the age of 50 years. This means that the values of each series were divided by the value at the age of 50 from the same series. The normalized values are expressed as relative heights: heights in relation to the height at 50 years. In this manner all series were combined at the age of 50 without disturbing the original shape of the curves. These normalized series were then divided into groups according to the values obtained at the age of 120 (*fig. 1*). The difference between the upper and the lower limit of each group was 7 percent. The lower limit of the first group was 1.00. None of the data fell within the first two groups. Thus, each group included series with identical or nearly identical height values at two ages instead of one. Mean growth values were calculated for all series within each group. These mean values represent what has been defined as a type of height growth (*table 1*).

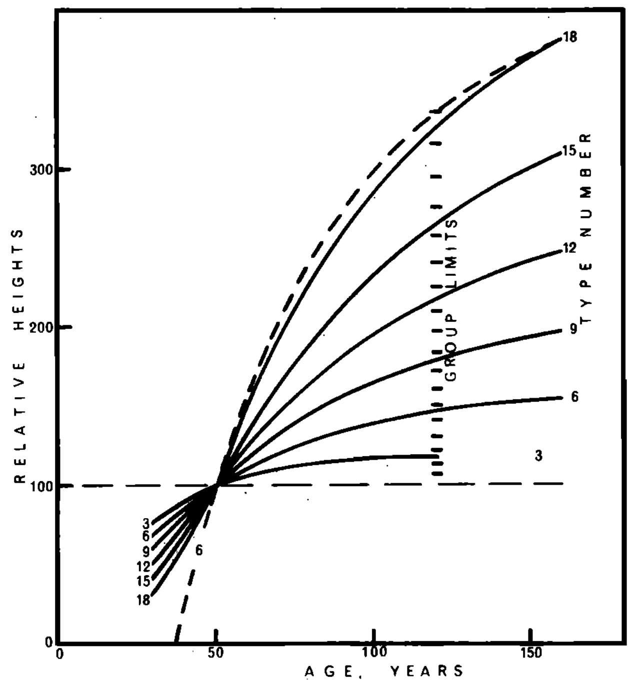

Figure 1. Construction of height growth types.

Table 1. Height growth types (height value in 50 years set at 100).

Age, years	Type number 3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
30	76	73	71	68	65	62	58	55	52	49	46	43	40	37	34	32
40	90	88	87	86	84	82	80	78	76	74	72	70	68	66	64	62
50	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
60	107	108	110	111	113	115	117	120	122	124	127	130	133	137	141	145
70	111	114	117	120	124	128	132	137	141	146	152	158	165	173	181	190
80	113	118	122	127	133	138	145	151	158	165	173	182	192	203	215	228
90	115	121	126	133	140	147	155	163	172	182	191	203	215	229	244	260
100	116	123	130	138	146	155	164	174	184	196	207	221	235	251	268	286
110	117	125	133	142	151	161	172	183	194	208	221	236	252	270	288	308
120	118	127	136	145	155	166	178	190	203	218	233	249	267	286	306	327
130		128	138	148	159	170	183	196	211	227	243	261	280	299	321	343
140		129	139	150	162	174	188	202	218	235	252	271	291	311	334	357
150			141	152	165	178	192	207	224	242	260	279	300	322	346	370
160				154	167	181	196	212	229	248	266	287	308	331	356	382

It is obvious that the set of height growth types is polymorphic since the curves are not proportional. If we rotate type 6 about its point of intersection with another type, 18 for instance, we will not be able to combine these types at all ages (broken line on *figure 1* depicts rotated type 6). The point of inflection is found after age 30 for types 12 to 18.

Our assumption that two points describe growth is not restricted to height growth; it is also relevant to other variables of stand growth. Therefore the same procedure with slight modification was applied to other variables. For both the number of trees per unit area and the form factor, values at the age of 100 were taken as 1.00 and group division was carried out at the age of 40. Differences between group limits were 5 percent for diameter and form factor, 7 percent for number of trees, 12 percent for basal area, and 15 percent for volume.

The crucial point was to calculate the accuracy with which all the other values of a given variable could be determined from its two known values. Statistical analysis of all the data showed that the average standard deviations within 30-160 years amounted to 2.4, 3.2, 4.5, 1.0, 6.9, and 5.9 percent of the mean values for height, diameter, basal area, form factor, number of trees, and volume, respectively. Similar results were obtained from 114 permanent sample plots where the standard deviation for diameter was 3.8 percent. This level of accuracy may be regarded as satisfactory since these deviations are within the error of practical determination of the respective variables in the field. Therefore, it is unnecessary to apply three, four, and more points. Two is the necessary and sufficient number. These growth types work equally well for all studied species. Separate calculation of types for individual species yielded only a slight increase in accuracy (for example, 0.9 percent for basal area).

Hence if different stands have the same growth values at any two ages, the values for these stands will also be the same at any other age within the limits of practical determination. This concept, being a generalization of empirical data, may be considered to be a principle: the principle of two points. The principle represents the simplest case of polymorphy in growth curves and can be defined in another way: growth curves may intersect not more than once. Since 1963 the principle has been developed for various forest variables in Russia (Zeide 1967; 1968a, b; 1971) and is presently becoming a routine method of forest studies there (Salikov 1974, Zagreeva 1974, Shkunov and Salikov 1975, Zagreev 1976).

Application of the Two-Point Principle

The main practical application of the two-point principle is apparent: to have the entire growth curve one has to find only two points and not a dozen as is usually done. All other points for any growth variable can be easily calculated.

For example, let us suppose that the average height of a certain stand was 69 feet at the age of 100 years and 41 feet at 50 years. The calculation includes the following steps:

1. Find the ratio between our heights:
$$\text{ratio} = \frac{69}{41} = 1.683.$$
2. Find the closest height growth type. The above ratio lies between 1.64, which is the ratio of appropriate values for type 9 (from *table 1*), and 1.74, the ratio for type 10. By interpolation the ratio 1.683 corresponds to the type 9.43.
3. Calculate type 9.43 for each age by interpolation of adjacent types 9 and 10. The result is provided in *table 2*.
4. Convert the type's values to heights. Multiply the values of type 9.43 by 41/100 or by 69/168.3, where 100 and 168.3 are the values of type 9.43 at 50 and 100 years respectively.

The same calculation is applicable to other growth variables. As the initial values one can take any pair with a reasonable time span between them (not less than 30 years).

In *table 2* calculated heights are compared with heights from the ponderosa pine (*Pinus ponderosa* Laws.) yield table from which the two initial values of 41 and 69 (at 50 and 100 years) were taken. The yield table was prepared by Myers et al. (1976) for managed diseased stands thinned initially (after attaining average stand diameter of 10 inches) to 120 square feet of basal area per acre and subsequently thinned to 90 square feet. Infestation of dwarf mistletoe (*Arceuthobium vaginatum* subsp. *cryptopodum* (Engelm.) Hawksw. & Wiens) starts at age 10. From *table 2* it is evident that even such stands can be satisfactorily described by growth types. Nevertheless a sharp change in conditions, such as sudden drainage, may appreciably alter the curve. The types might be useful in this case if they could be determined before and after the drainage; the difference then would provide a quantitative measure of the drainage effect.

Discussion

Standardization of growth curves does not neglect differences due to inheritance, site, or human activity. If conditions are different, they must be reflected by growth values at one age and all the more at two ages. The growth types are a convenient means to describe these differences. For instance, it was found that larch (*Larix europaea* DC.) occupies diameter growth types from 8 to 15 and practically does not intersect with fir (*Abies alba* Mill.) occupying diameter types 15-27.

Along with such technical application which enables us to simplify growth studies in any area, we can draw from this principle some general conclusions on growth modeling. The very fact that we have so many

Table 2. Comparison between calculated and actual height growth of ponderosa pine.

Age, years	Height growth type 9.43	Height: Calculated	Height: Yield table	Difference
		........Feet........		Feet
30	56.7	23.2	24.0	−0.8
40	79.1	32.4	32.0	.4
50	100.0	41.0	41.0	0
60	118.3	48.5	47.0	1.5
70	134.2	55.0	54.0	1.0
80	147.6	60.5	60.0	0.5
90	158.4	64.9	64.0	0.9
100	168.3	69.0	69.0	0
110	176.7	72.4	72.5	−0.1
120	183.2	75.1	76.0	−0.9
130	188.6	77.3	79.0	−1.7

formulas for the same relation (for instance, height/age) implies that they are of restricted value. The accuracy of formulas depends on the number of parameters rather than on the particular form of the equation. Such a situation allows us to shift the discussion from particular formulas to the more general problem of the number of parameters needed to describe stand growth. Such formulas usually contain three or more parameters. The principle of two points enables us to notice that the number is superfluously high. One can expect to find that only two parameters are independent; others should be a combination of these two.

This prediction was checked against parameters of the exponential-monomolecular function as computed by Lundgren and Dolid (1970) for eleven Lake States timber species. The function has three parameters: b_1, b_2, and b_3. The calculations showed that a high correlation exists between them: R^2 (b_1; b_2, b_3)=0.99. Hence, at least one parameter is a function of others. We can easily arrive at the same conclusion by considering Beck's (1971) calculations. He found that one of three parameters simply does not vary with site index. More substantial proof of the prediction is presented in a recent work of Karish and Borden (1976). Applying five widely adopted nonlinear growth functions, they found that "only two linear combinations are required to account for almost all of the effects of the Chapman-Richards' four parameters and the three parameters of the other models. In other words, it appears possible to reduce the dimensionality of the models to two, and obtain an altered model which would be as effective for fitting the data. . . . Hence it appears that most biological interpretations of growth model parameters are of questionable validity."

Shortcomings of the presentation of growth by formulas are rarely discussed. Unlike equations for physical laws, functions used to describe tree or stand growth do not reflect the essence of the growth. Usually we use a formula if its shape resembles the growth curve's shape, but this resemblance seldom means identity. Until we discover a genuine growth function, it would probably be safer to rely on tables. The two-point principle can be used in any form—formula, table, or graph. It is particularly valuable because it allows us to avoid use of questionable formulas and provides a basis for construction of growth types through generalization of yield tables. The types may be easily stored in a computer memory and used for calculations just as well as formulas.

The principle of two points and especially the growth types have some limitations. Since the types were constructed from a restricted number of yield tables for conifer and hardwoods, they probably do not encompass stands making very slow or very fast growth. The types do not cover all tree species. Attempts to prepare sets of growth types for fast-growing species have already been reported (Salikov 1974, Shkunov and Salikov 1975). A small increase in accuracy may be gained by constructing types for each species separately as Zagreeva (1974) has done for spruce stands.

The two-point principle by no means replaces the one-point approach. Because of its extreme simplicity, the latter remains an indispensable tool. Perhaps the most important practical consideration is that the use of growth types can provide the shortest method for determining the limits within which the one-point principle and conventional sets of site index curves will work properly. The sets themselves may be quickly prepared on the basis of two height values. Both approaches are complementary: the one-point principle is the simplest way of growth estimation but has limited application; the two-point principle so far has no geographical or species restrictions though it is not easy to find the second point on the same growth curve. ■

Literature Cited

BAILEY, R. L., and J. L. CLUTTER. 1974. Base-age invariant polymorphic site curves. For. Sci. 20:155-159.

BARNES, G. H. 1962. Yield of even-aged stands of western hemlock. USDA Tech. Bull. 1273, 52 p.

BECK, D. E. 1971. Height-growth patterns and site index of white pine in the southern Appalachians. For. Sci. 17:252-260.

BULL, H. 1931. The use of polymorphic curves in determining site quality in young red pine plantations. J. Agric. Res. 43:1-28.

GRAVES, H. S. 1906. Forest Mensuration. John Wiley & Sons, New York. 458 p.

KARISH, J. F., and F. Y. BORDEN. 1976. Parameter correlation effects in non-linear mathematical models for biological growth. Penn. State Univ., School For. Resour., Res. Briefs 10:11-15.

LUNDGREN, A. L., and W. A. DOLID. 1970. Biological growth functions describe published site index curves for Lake States timber species. USDA For. Serv. Res. Pap. NC-36, 9 p.

MEYER, W. H. 1937. Yield of even-aged stands of Sitka spruce and western hemlock. USDA Tech. Bull. 544, 86 p.

MYERS, C. A., C. B. EDMINSTER, and F. G. HAWKSWORTH. 1976. SWYLD2: Yield tables for even-aged and two-storied stands of southwestern ponderosa pine, including effects of dwarf mistletoe. USDA For. Serv. Res. Pap. RM-163, 25 p.

SALIKOV, N. Y. 1974. [Series of height growth types for birch stands.] P. 208-213 *in* Materiali nauch. konf. molodih uchenih N3. VNII lesovodstva u mekhanizatsii les. kh-va. Pushkino (in Russian).

SHKUNOV, V. A., and N. Y. SALIKOV. 1975. [Compiling yield tables for modal birch stands.] Lesn. Khozyaistvo N8, 60-62 (in Russian).

STAGE, A. R. 1963. A mathematical approach to polymorphic site index curves for grand fir. For. Sci. 9:167-180.

TURIN, A. V. 1913. [Study of growth of normal pine stands in Arkhangelsk region.] Trudi po lesnomu opitnomu delu v Rossii. N45 Lesnoi Department Sanct-Petersburg, 135 p. (in Russian).

ZAGREEV, V. V. 1976. [Typification and standardization of natural growth series.] Lesn. Khozyaistvo, N11, 69-74 (in Russian).

ZAGREEVA, A. I. 1974. [Developing standard site index curves for spruce stands.] P. 202-207 *in* Materiali nauch. konf. molodih uchenih N3. VNII lesovodstva u mekhanizatsii les. kh-va. Pushkino (in Russian).

ZEIDE, B. B. 1967. [Unification of diameter growth series.] P. 303-313 *in* Voprosy drevesnogo prirosta v lesoustroystve. Ed. V. Antanaitis. Litovskaya Selskokhozyaistvennaya Akademia, Kaunas. 387 p. (in Russian).

ZEIDE B. B. 1968a. [Standardization of height-growth data.] P. 128-136 *in* Issledovaniya po lesnoi taksatzii i lesoystroistvu VNILM. Ed. N. P. Anuchin. Lesn. Prom., Moscow, 223 p. (in Russian).

ZEIDE, B. B. 1968b. [Standardization of forest stand growth parameters.] Lesn. Khozyaistvo, N10, 54-57 (in Russian).

ZEIDE, B. B. 1971. [On standardization principles used in study of stand growth.] P. 45-65 *in* Nauchnie trudi, N3. Ed. M. N. Ribakov. Vsesoyuzny n.-i. Institut Standardizatsii, Moscow. 212 p. (in Russian).

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